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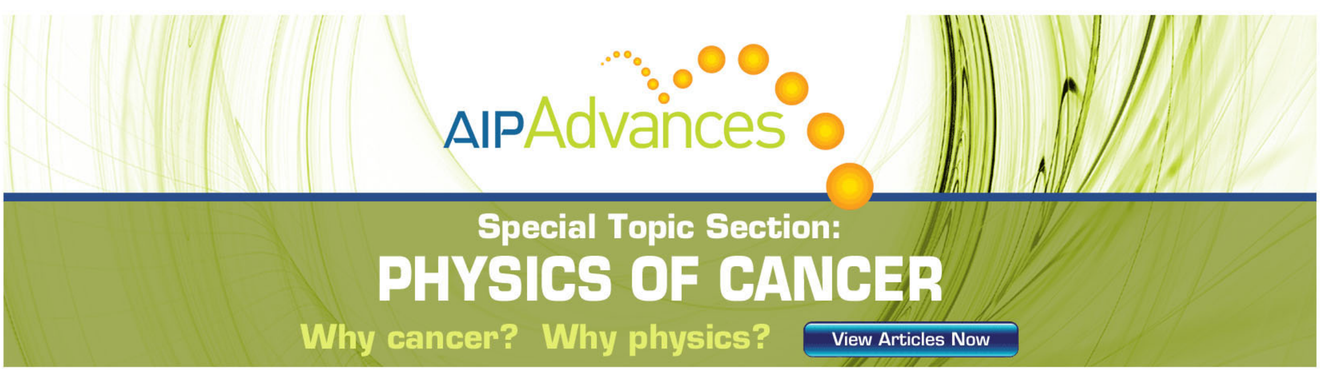

PHYSICS OF PLASMAS **19**, 122108 (2012)

Spontaneous electromagnetic fluctuations in unmagnetized plasmas. III. Generalized Kappa distributions

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In the first two papers of this series, the general expressions for the spontaneous fluctuations spectra (electric and magnetic field, charge and current densities) from uncorrelated plasma particles are derived and illustrated for a Maxwellian (relativistic or nonrelativistic) plasma close to thermal equilibrium. In this paper, the results are illustrated for the nonideal case of a plasma out of thermal equilibrium and described by the generalized Kappa (power-law) particle distribution function in the nonrelativistic limit. The suprathermal fluctuations of weakly amplified modes and aperiodic modes are provided. Thus, it is shown for the first time the existing finite level of noncollective fluctuations, which are particularly important in the context of plasma fluctuations (collective or noncollective) as the best agent in the energy dissipation and transfer to suprathermal populations. The results obtained in the first paper for an equilibrium plasma are recovered only in the limit of a very large power index $\kappa \to \infty$. © *2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4769308]

I. INTRODUCTION

General expressions for the electromagnetic fluctuation spectra (electric and magnetic field, charge and current densities) have been calculated in the first two papers[1,2] [hereafter called Paper I and Paper II] of this series, on the basis of the Klimontovich and Maxwell equations. These results have particularly been illustrated for an isotropic Maxwellian particle distribution function in relativistic[2] or nonrelativistic[1,3] plasmas. It has originally been shown the existence of a finite level of spontaneously emitted aperiodic fluctuations of electrostatic and magnetic field, in full dependence on the wavenumber and the positive growth rate.

In this paper we evaluate the spontaneously emitted electromagnetic fluctuations for the less ideal case of a nonequilibrium Kappa distributed plasma. These investigations are particularly important given the implications of the generalized power-law (Kappa-like) distribution functions widely exploited to model plasma out of thermal equilibrium.[4–11]

According to the observations, suprathermal populations are ubiquitous in the solar wind and magnetosphere and are expected to exist in any low-density plasma in space (where Coulomb collisions are sufficiently rare).[4,6,11] Such deviations from thermal equilibrium are modeled by the Kappa distribution functions and are well explained by the action of plasma wave fluctuations as the best agent in the conversion and transport of the free energy to high energy tails of suprathermal distributions:[7–9,11–13] wave-particle interactions replace binary collisions and enhance dispersive effects heating plasma particles. Thus, the distribution functions of Kappa-type have been invoked to generalize the notion of equilibrium and entropy for the poor-collisional (or even collisionless) plasmas, out of thermal (Maxwellian) equilibrium, but containing a fully developed field of correlated wave fluctuations in quasistationary equilibrium with plasmaparticles.[14,15]

In turn, the general plasma dynamics, dispersion properties, and stability are also altered by the presence of suprathermal populations. Theories developed in the last decades include both isotropic or anisotropic (bi-Kappa) models and have shown that properties of the collective plasma wave fluctuations are highly dependent on the value of the power index κ, leading to significant deviations from the well-known standard features of the Maxwellian distributions (see the results reviewed by Hellberg *et al.*[10] and by Pierrard and Lazar[11]). Moreover, it has been proposed to evaluate the electron (noncollective) quasithermal noise and develop a powerful technique for *in situ* space plasma diagnostics[16–18] as it is less sensitive to the spacecraft perturbations than particle detectors. The first analysis of the solar wind data from the radio receiver of Ulysses mission has recently reported good fittings with calculations of the plasma quasithermal noise for a generalized Kappa distribution function.[17,19]

The organization of this paper is as follows: in Sec. II we start from the general expression obtained in the first two papers with the derivation of the electric and magnetic field fluctuation spectra, the perpendicular current density, and the charge density, for Kappa power-law distribution functions (in the nonrelativistic limit). First we calculate the form factors and particularize for weakly amplified and weakly

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propagating (or aperiodic) fluctuations. Correspondingly, the spontaneously emitted field fluctuations are derived and displayed for comparison with the limiting case of Maxwellian plasmas analyzed in Paper I. Finally, in Sec. III we summarize the results.

II. SPONTANEOUS FLUCTUATIONS OF A KAPPA DISTRIBUTED PLASMA

A. Basic equations

We start from the generalized expressions for the electric and magnetic field fluctuations derived in Paper I and corrected in Paper II, see, e.g., Eq. (1), which read (notation identical to previous papers)

$$\begin{pmatrix} \langle \delta E_\parallel^2 \rangle_{k,\omega} \\ \langle \delta E_\perp^2 \rangle_{k,\omega} \\ \langle \delta B_\perp^2 \rangle_{k,\omega} \end{pmatrix} = \sum_a \frac{\omega_{p,a}^2 m_a}{4\pi^3 k^2} \begin{pmatrix} \dfrac{K_\parallel(k,\omega)}{|\omega \Lambda_L(\vec{k},\omega)|^2} \\ \dfrac{K_\perp(k,\omega)}{|\omega \Lambda_T(\vec{k},\omega)|^2} \\ \dfrac{c^2 k^2 K_\perp(k,\omega)}{|\omega^2 \Lambda_T(\vec{k},\omega)|^2} \end{pmatrix}. \quad (1)$$

Fully relativistic expressions of the form factors $K_{\parallel,\perp}$ and the dispersion functions $\Lambda_{L,T}$ are derived in Paper II for an *arbitrary* number of species a of charged particles and *arbitrary* forms of their (isotropic) velocity distributions. These read as follows:

$$\begin{aligned} K_\parallel(k,\omega) &= kc\Re[-\imath z] - kc(m_a c)^3 N_a \int_1^\infty dE f_a(E) \\ &\quad \times E^2 \Re[\imath z^2 J(E,z)] \\ K_\perp(k,\omega) &= kc\Re[-\imath z] - kc(m_a c)^3 N_a \int_1^\infty dE f_a(E) \\ &\quad \times \Re\{\imath[E^2(1-z^2)-1]J(E,z)\}, \end{aligned} \quad (2)$$

$$\begin{aligned} \Lambda_L(\vec{k},\omega) &= 1 - \sum_a \frac{2N_a \omega_{p,a}^2 (m_a c)^3}{k^2 c^2} \left[\int_1^\infty dE E\sqrt{E^2-1} \right. \\ &\quad \left. \times \frac{\partial f_a(E)}{\partial E} + \frac{z}{2} \int_1^\infty dE E^2 \frac{\partial f_a(E)}{\partial E} J(E,z) \right], \\ \Lambda_T(\vec{k},\omega) &= 1 - \frac{1}{z^2} + \sum_a \frac{N_a \omega_{p,a}^2 (m_a c)^3}{k^2 c^2} \left[\int_1^\infty dE E \right. \\ &\quad \times \sqrt{E^2-1} \frac{\partial f_a(E)}{\partial E} - \frac{1}{2z} \int_1^\infty dE \\ &\quad \left. \times [E^2(1-z^2)-1] \frac{\partial f_a(E)}{\partial E} J(E,z) \right], \end{aligned} \quad (3)$$

where $E = [1 + p^2/(m_a c)^2]^{1/2}$ is the relativistic Lorentz factor, $z = \omega/(kc)$,

$$N_a = \left[2(m_a c)^3 \int_1^\infty dE E\sqrt{E^2-1} f_a(E) \right]^{-1},$$

and

$$J(E,z) = \int_{-\sqrt{1-E^{-2}}}^{\sqrt{1-E^{-2}}} \frac{dt}{t-z} + 2\pi\imath\sigma$$

with $\sigma = 0, 1, 2$ for the imaginary frequency $\Im(\omega) \equiv \gamma > 0$, $0, < 0$, respectively. Directions parallel (subscript "$\parallel$") and perpendicular (subscript "$\perp$") are considered with respect to the direction of the wave-vector $\vec{k} = k\hat{z}$.

We also remember that Eq. (1) provides for the charge and current density fluctuations[2]

$$\begin{pmatrix} \langle \delta\rho^2 \rangle_{k,\omega} \\ \langle \delta J_\parallel^2 \rangle_{k,\omega} \\ \langle \delta J_\perp^2 \rangle_{k,\omega} \end{pmatrix} = \sum_a \frac{\omega_{p,a}^2 m_a}{2(2\pi)^5} \begin{pmatrix} \dfrac{K_\parallel(k,\omega)}{|\omega \Lambda_L(\vec{k},\omega)|^2} \\ \dfrac{K_\parallel(k,\omega)}{|k \Lambda_L(\vec{k},\omega)|^2} \\ \dfrac{\left[1 + \dfrac{c^4 k^4}{|\omega|^4}\right] K_\perp(k,\omega)}{|k \Lambda_T(\vec{k},\omega)|^2} \end{pmatrix}. \quad (4)$$

B. The Kappa distribution function

Now we introduce the isotropic (nonrelativistic) Kappa distribution function[20,21]

$$f_\kappa(v) = \frac{1}{(\pi\kappa w_\kappa^2)^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma\left(\kappa - \dfrac{1}{2}\right)} \left(1 + \frac{v^2}{\kappa w_\kappa^2}\right)^{-(\kappa+1)}, \quad (5)$$

$$w_\kappa^2 = \left(1 - \frac{3}{2\kappa}\right)\left(\frac{2k_B T}{m}\right), \quad (6)$$

which is a power-law generalization of the equilibrium Maxwellian, introduced to describe nonequilibrium plasmas with suprathermal velocity distributions (Fig. 1) observed in space.[4,6] Distributions measured in the solar wind provide the best fits for a power-index in the range κ =2–6, which is sufficiently large to avoid the critical

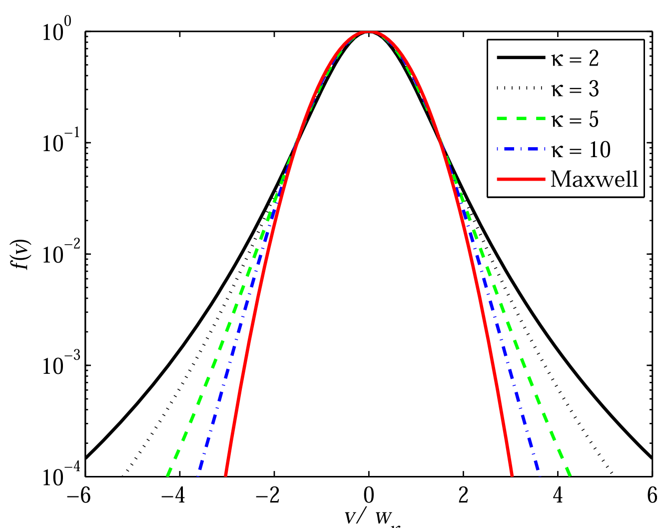

FIG. 1. Kappa distribution functions approach a thermal Maxwellian core at small velocities, less than thermal speed $v \ll w_\kappa$, but enhances showing suprathermal tails at high energies, $v > w_\kappa$.

value $\kappa_c \equiv 3/2 < \kappa$, where the distribution function (5) collapses and the (equivalent) temperature is not defined. Here w_κ is the effective thermal velocity of the charged particles, m is the mass, n is the number density, T is the effective temperature, and $\Gamma(x)$ is the Gamma function. In the limit of a large power index, $\kappa \to \infty$ ($w_{\kappa\to\infty} = v_T = \sqrt{2k_B T/m}$), the power-law distribution function reduces to a Maxwellian, $f_\kappa(v) \to f_M(v)$ (red line in Fig. 1).

C. Form factors

In the limit of nonrelativistic plasma temperatures ($E \to 1$) the form factors in Eq. (2) become

$$\begin{pmatrix} K_\parallel(k,\omega) \\ K_\perp(k,\omega) \end{pmatrix} = k^2 \Re \int d^3v \, \frac{f_a(\vec{v})}{\gamma + \imath(\vec{k}\cdot\vec{v} - r)} \begin{pmatrix} v_\parallel^2 \\ v_\perp^2 \end{pmatrix},$$

which agree with the form derived in Paper I and, for the distribution function (5), transform to

$$K_\parallel(k,\omega) = C_a k^2 \Re \left[i \int_{-\infty}^{+\infty} dv_z \frac{v_z^2}{\omega - kv_z} \int_{-\infty}^{+\infty} dv_x dv_y \times \left(1 + \frac{v_x^2 + v_y^2 + v_z^2}{\kappa w_a^2}\right)^{-(\kappa+1)} \right] \tag{7}$$

and

$$K_\perp(k,\omega) = C_a k^2 \Re \left[i \int_{-\infty}^{+\infty} \frac{dv_z}{\omega - kv_z} \int_{-\infty}^{+\infty} dv_x dv_y \times (v_x^2 + v_y^2) \left(1 + \frac{v_x^2 + v_y^2 + v_z^2}{\kappa w_a^2}\right)^{-(\kappa+1)} \right], \tag{8}$$

with the normalization constant from Eq. (5)

$$C_a = \frac{1}{(\pi w_a)^{3/2}} \frac{\Gamma(\kappa)}{\kappa^{1/2}\Gamma(\kappa - 1/2)}. \tag{9}$$

For simplicity, we omit the subscript κ but keep the subscript a indicating contributions from different plasma species (e.g., $a = e$, for electrons, and $a = p$ for protons).

After integrations we obtain the form factors

$$K_\parallel(k,\omega) = -\Re\left\{ i\omega \left[1 + \frac{\omega}{kw_a} Z_\kappa^0\left(\frac{\omega}{kw_a}\right) \right] \right\} \tag{10}$$

and

$$K_\perp(k,\omega) = -\left(1 - \frac{1}{\kappa}\right) kw_a \Re \times \left\{ iZ_\kappa^0\left(\frac{\omega}{kw_a}\right) + \frac{\omega}{\kappa kw_a}\left[1 + \frac{\omega}{kw_a} Z_\kappa^0\left(\frac{\omega}{kw_a}\right)\right] \right\}, \tag{11}$$

in terms of the modified dispersion function (A5)–(A7). In the limit of a very large power index $\kappa \to \infty$ the form factors reduce exactly to the Maxwellian expressions derived in Paper I (see Eqs. (56) and (57)).

In the case of weakly amplified fluctuations $\omega = r + i\gamma$ with $r \gg \gamma \to 0$, if we choose a small value for the power index $\kappa = 2$ and apply Eq. (B8), see Appendix B, the form factors reduce to

$$K_\parallel(k,\omega) = \frac{4\sqrt{2}rx}{(x^2+2)^2} \tag{12}$$

and

$$K_\perp(k,\omega) = \frac{kw}{2} \frac{(x^2-2)x + 4\sqrt{2}}{(x^2+2)^2}, \tag{13}$$

whereas in the case of aperiodic fluctuations with $\gamma \gg r \to 0$, we apply Eq. (B7) and find

$$K_\parallel(k,\omega) = \frac{2\gamma}{(y+\sqrt{2})^2} \tag{14}$$

and

$$K_\perp(k,\omega) = \frac{kw}{2} \frac{y + 2\sqrt{2}}{(y+\sqrt{2})^2}. \tag{15}$$

In Appendix B, we have introduced $x = r/(kw)$ and $y = \gamma/(kw)$, where w is the same thermal velocity defined in Eq. (6), but the subscript a is omitted for convenience.

D. Dispersion functions

Again with the distribution function (5), we obtain in Appendix A the nonrelativistic dispersion relations for the longitudinal waves

$$\Lambda_L = 1 + \sum_a \frac{2\omega_{p,a}^2}{k^2 w_a^2}\left[\left(1 - \frac{1}{2\kappa}\right) + \frac{\omega}{kw_a} Z_\kappa\left(\frac{\omega}{kw_a}\right)\right] \tag{16}$$

and for the transverse waves

$$\Lambda_T = 1 - \frac{c^2k^2}{\omega^2} + \sum_a \frac{\omega_{p,a}^2}{\omega kw_a} Z_\kappa^0\left(\frac{\omega}{kw_a}\right), \tag{17}$$

in terms of the modified plasma dispersion function (A5)–(A7). In the limit of a very large $\kappa \to \infty$ we find exactly the Maxwellian dispersion function, see, for instance, Eqs. (54) and (55) in Paper I.

For the same value of the power index $\kappa = 2$, we apply Eqs. (B6) and (B8) to describe the weakly amplified fluctuations ($\gamma \to 0$ and $y \to 0$) by

$$\Lambda_L = 1 + \sum_a \frac{\omega_{p,a}^2}{r^2}\left[\frac{x_a^2(12 - x_a^4 - 21x_a^2)}{(x_a^2+2)^3} + \imath \frac{16\sqrt{2}x_a^3}{(x_a^2+2)^3}\right] \tag{18}$$

and for the transverse waves

$$\Lambda_T = 1 - \frac{c^2k^2}{r^2} + \sum_a \frac{\omega_{p,a}^2}{r^2}\left[\frac{x_a^2(x_a^2+6)}{(x_a^2+2)^2} - \imath\frac{4\sqrt{2}x_a}{(x_a^2+2)^2}\right], \quad (19)$$

whereas for the aperiodic fluctuations ($r \to 0$ and $x \to 0$) we apply Eqs. (B5) and (B7) and find

$$\Lambda_L = 1 + \sum_a \frac{\omega_{p,a}^2}{k^2 w_a^2}\frac{y_a+12}{(y_a^2+\sqrt{2})^3} \quad (20)$$

and

$$\Lambda_T = 1 - \frac{c^2k^2}{\omega^2} - \sum_a \frac{\omega_{p,a}^2}{\gamma k w_a}\frac{y_a+2\sqrt{2}}{(y_a+\sqrt{2})^2}. \quad (21)$$

E. Electromagnetic fluctuations

Collecting all the terms we obtain for the electric and magnetic field fluctuation spectra (1)

$$\langle\delta E_\parallel^2\rangle_{k,\omega} = -\sum_a \frac{\omega_{p,a}^2 m_a}{4\pi^3 k^2}\frac{1}{r^2+\gamma^2} \times \frac{\Re\left\{\imath\omega\left[1+\frac{\omega}{kw_a}Z_\kappa^0\left(\frac{\omega}{kw_a}\right)\right]\right\}}{\left|1+\sum_a \frac{2\omega_{p,a}^2}{k^2w_a^2}\left[1-\frac{1}{2\kappa}+\frac{\omega}{kw_a}Z_\kappa\left(\frac{\omega}{kw_a}\right)\right]\right|^2}, \quad (22)$$

$$\langle\delta E_\perp^2\rangle_{k,\omega} = -\sum_a \frac{\omega_{p,a}^2 m_a w_a}{4\pi^3 k}\left(1-\frac{1}{\kappa}\right)\frac{1}{r^2+\gamma^2} \times \frac{\Re\left\{\imath Z_\kappa^0\left(\frac{\omega}{kw_a}\right)+\frac{\omega}{\kappa k w_a}\left[1+\frac{\omega}{kw_a}Z_\kappa^0\left(\frac{\omega}{kw_a}\right)\right]\right\}}{\left|1-\frac{k^2c^2}{\omega^2}+\sum_a\frac{\omega_{p,a}^2}{\omega k w_a}Z_\kappa^0\left(\frac{\omega}{kw_a}\right)\right|^2}, \quad (23)$$

and

$$\langle\delta B_\perp^2\rangle_{k,\omega} = \frac{k^2c^2}{r^2+\gamma^2}\,\langle\delta E_\perp^2\rangle_{k,\omega}. \quad (24)$$

These can be used to find the charge and current density fluctuations (4)

$$\langle\delta\rho^2\rangle_{k,\omega} = \frac{k^2}{(4\pi)^2}\,\langle\delta E_\parallel^2\rangle_{k,\omega}, \quad (25)$$

$$\langle\delta J_\parallel^2\rangle_{k,\omega} = \frac{|\omega|^2}{(4\pi)^2}\,\langle\delta E_\parallel^2\rangle_{k,\omega},$$
$$\langle\delta J_\perp^2\rangle_{k,\omega} = \frac{|\omega|^2}{(4\pi)^2}\left[1+\frac{k^4c^4}{|\omega|^4}\right]\langle\delta E_\perp^2\rangle_{k,\omega}. \quad (26)$$

Expressions derived in Paper I for these fluctuations in Maxwellian plasmas (closed to thermal equilibrium) are exactly recovered in the limit of a large power index $\kappa \to \infty$. We have however to remember that the same correction factor $|\omega|^{-2}$ for the electrostatic fluctuations in Eq. (1) must be applied to Eqs. (63) and (66) in Paper I.

F. Suprathermal aperiodic fluctuations

The aperiodic ($r=0$) collective modes of Weibel-type[22–24] with a positive growth rate ($0 < \gamma \ll kw_a$) are driven by the anisotropic plasma distributions. In Paper I it was shown that non-collective fluctuations with $r=0$ also result in isotropic distributions at thermal (Maxwellian) equilibrium. Here, the investigation is further generalized to an isotropic Kappa power-law distribution function.

Thus, referring to the limit of Eqs. (22) and (24) as thermal aperiodic fluctuations ($r, x \to 0$) and using the approximations (B5) and (B7), we obtain for an isotropic $\kappa = 2$ distribution

$$\langle\delta E_\parallel^2\rangle_{k,\gamma} = \frac{\frac{1}{2\pi^3k^2\gamma}\sum_a\frac{\omega_{p,a}^2 m_a}{(y_a+\sqrt{2})^2}}{\left[1+\sum_a\frac{\omega_{p,a}^2}{\gamma^2}\frac{y_a^2(y_a+3\sqrt{2})}{(y_a+\sqrt{2})^3}\right]^2}, \quad (27)$$

$$\langle\delta E_\perp^2\rangle_{k,\gamma} = \frac{\sum_a\frac{\omega_{p,a}^2 m_a}{8\pi^3k^2\gamma}\frac{y_a+2\sqrt{2}}{y_a(y_a+\sqrt{2})^2}}{\left[1+\frac{k^2c^2}{\gamma^2}+\sum_a\frac{\omega_{p,a}^2}{\gamma^2}\frac{y_a(y_a+2\sqrt{2})}{(y_a+\sqrt{2})^2}\right]^2}, \quad (28)$$

and

$$\langle\delta B_\perp^2\rangle_{k,\gamma} = \frac{k^2c^2}{\gamma^2}\,\langle\delta E_\perp^2\rangle_{k,\gamma}, \quad (29)$$

where, to remember, $x_a = r/(kw_a) = 0$ and $y_a = \gamma/(kw_a)$. In order to verify, the same expressions for the aperiodic fluctuations are obtained directly, using the approximations for the dispersion functions (20) and (21) and for the form factors (14) and (15).

Figure 2 displays by comparison the contour plots of the spontaneously emitted aperiodic electrostatic fluctuations (27) in a Kappa distributed plasma ($\kappa = 2$) out of thermal equilibrium (top panel) and a thermal Maxwellian ($\kappa \to \infty$) plasma (bottom panel). In a Maxwellian distribution the highest fluctuation levels occur at small values of γ/ω_{pe} and large wave-number values (bottom panel). However, in the presence of suprathermal populations and a diminished thermal population, the quasi-thermal aperiodic noise is significantly reduced (top panel), at least one order of magnitude for the same growing time scale.

The aperiodic magnetic fluctuations (29) spontaneously emitted in the Kappa and Maxwellian plasmas are shown in Fig. 3, top panel and bottom panel, respectively. Because the data displayed in Fig. 4 from Paper I are affected by a sign

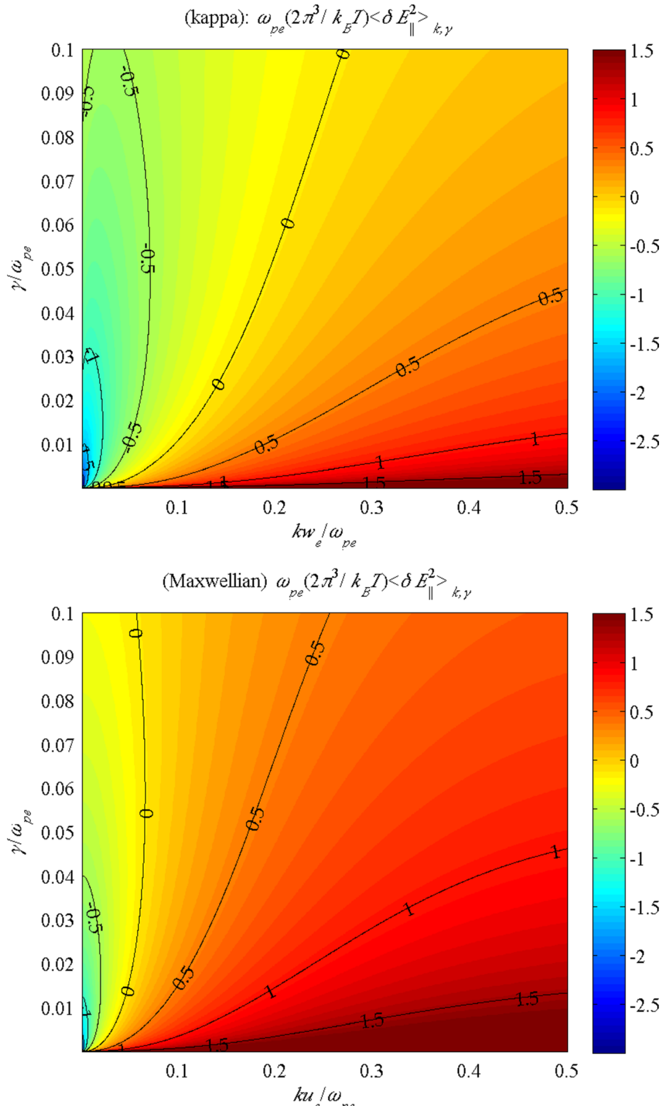

FIG. 2. Contour and color map plots of the spontaneously emitted aperiodic electrostatic field fluctuation in units of $k_BT_e/(2\pi^3)$ by contrast for a suprathermal Kappa-distributed plasma (top, $\kappa = 2$) and a thermal (Maxwellian) plasma (bottom). Equal electron and proton temperatures ($T_i = T_e$) and nonrelativistic thermal electron velocity $u_e = 10^{-3}$c are adopted.

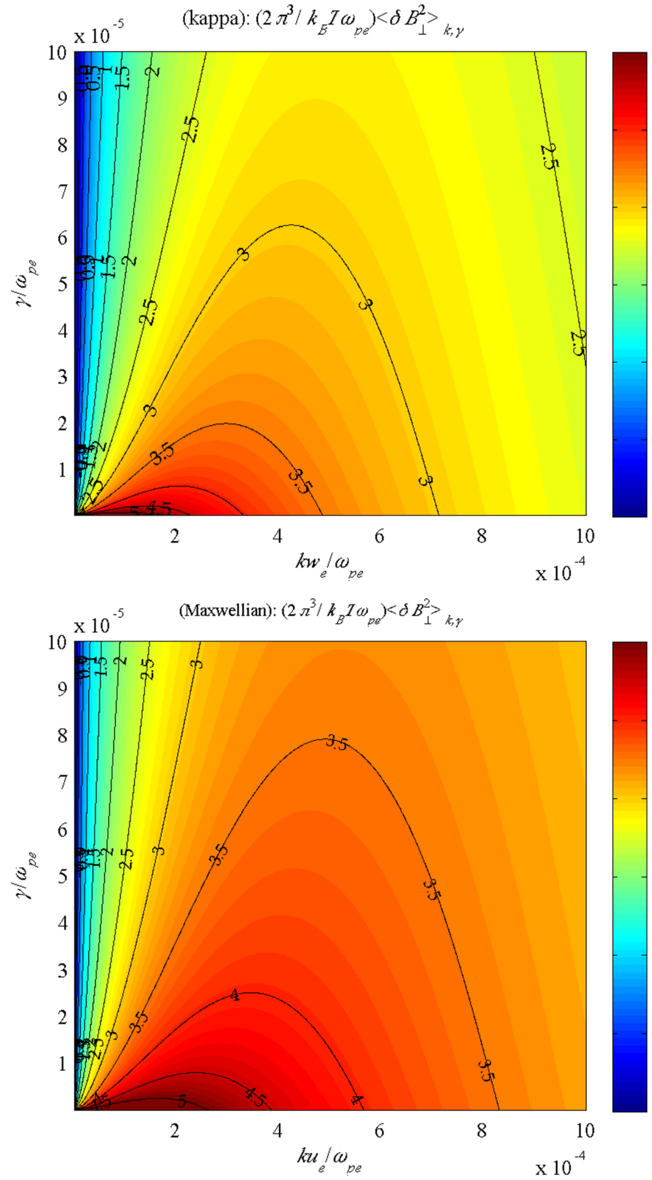

FIG. 3. Contour and color map plots of the spontaneously emitted aperiodic magnetic field fluctuation in units of $k_BT_e/(2\pi^3)$ by contrast for a suprathermal Kappa-distributed plasma (top, $\kappa = 2$) and a thermal (Maxwellian) plasma (bottom). Equal electron and proton temperatures ($T_i = T_e$) and the nonrelativistic thermal electron velocity $u_e = 10^{-3}$c are adopted.

error in the exponential function, here in Fig. 3 (bottom panel) we provide the correct contour and color map plots of the aperiodic magnetic fluctuations for the same characteristics of a Maxwellian plasma. These fluctuations correspond to the so called Weibel aperiodic solutions,[22–24] which fluctuate only in space but permanently grow or decrease in time depending on the sign of γ. Here, the highest fluctuation levels occur at small values of the wave-number $k \ll \omega_{pe}/w_e$ and the growth rate γ. Moreover, the same effect is observed in Kappa distributions, where for small values of the power index κ these levels are significantly inhibited (i.e., the highest aperiodic noise is obtained for a large $\kappa \to \infty$ distributed plasma closed to thermal equilibrium, e.g., bottom panel).

G. Suprathermal weakly amplified fluctuations

With the approximations (B6)–(B8), we immediately derive the nonrelativistic thermal fluctuation spectra for weakly amplified fluctuations in the limit $\gamma \to 0$ (or $y \to 0$)

$$\langle\delta E_\parallel^2\rangle_{k,r} = \sum_a \frac{\sqrt{2}\omega_{p,a}^2 m_a x_a}{\pi^3 k^2 r (x_a^2+2)^2} \times \left\{ \left[1 + \sum_a \frac{\omega_{p,a}^2}{r^2}\frac{x_a^2(12 - x_a^4 - 21x_a^2)}{(x_a^2+2)^3}\right]^2 + \left[\sum_a \frac{16\sqrt{2}\omega_{p,a}^2 x_a^3}{r^2(x_a^2+2)^3}\right]^2 \right\}^{-1}, \tag{30}$$

$$\langle \delta E_\perp^2 \rangle_{k,r} = \sum_a \frac{\omega_{p,a}^2 m_a}{8\pi^3 k^2 r} \frac{x_a(x_a^2-2)+4\sqrt{2}}{x_a(x_a^2+2)^2} \times \left\{ \left[1 - \frac{k^2c^2}{r^2} - \sum_a \frac{\omega_{p,a}^2 x_a^2(x_a^2+6)}{r^2(x_a+2)^2} \right]^2 + \left(\sum_a \frac{4\sqrt{2}\omega_{p,a}^2 x_a}{r^2(x_a+2)^2} \right)^2 \right\}^{-1}, \tag{31}$$

and

$$\langle \delta B_\perp^2 \rangle_{k,r} = \frac{k^2c^2}{r^2} \langle \delta E_\perp^2 \rangle_{k,r}. \tag{32}$$

These results are displayed in Figures 4 and 5 for a Kappa ($\kappa = 2$) distributed plasma (top panels) and for a Maxwellian ($\kappa \to \infty$) plasma (bottom panels). The electrostatic fluctuations (Fig. 4) seem to be markedly dependent on the power index κ, as well as on the wave-number and the range of frequency. At high frequencies corresponding to electron plasma oscillations, the peak of the quasithermal noise markedly depends on the distribution function. For a Kappa distribution ($\kappa = 2$) the peak moves to small wave-numbers, nearly at the plasma frequency. Measurements of the electrostatic noise using an antenna immersed in a Kappa-distributed plasma confirm this effect.[16] But this effect is not necessarily predicted by the linear dispersion theory of the collective Langmuir oscillations as their Landau damping in the presence of suprathermal populations is enhanced at small wave-numbers and becomes less important at larger wave-numbers.[21]

For lower frequencies, including the low-frequency fluctuations associated with ion-sound modes, the quasithermal noise is in general inhibited and depends mostly on the low-

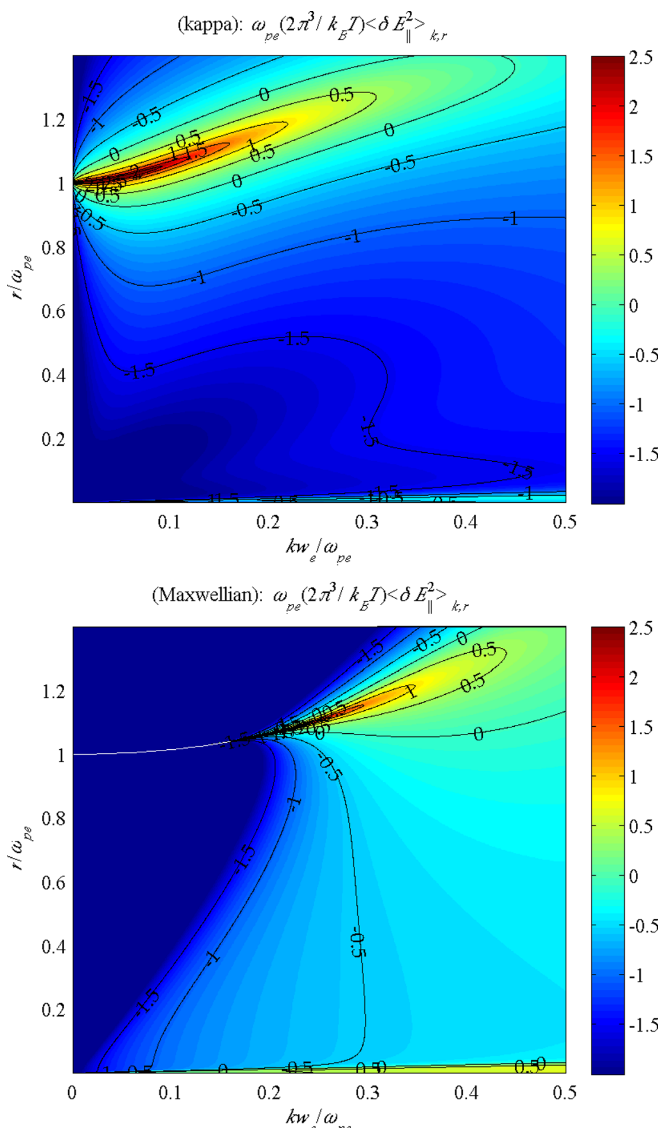

FIG. 4. Contour and color map plots of the spontaneously emitted weakly amplified electrostatic field fluctuation in units of $k_BT_e/(2\pi^3)$ by contrast for a suprathermal Kappa-distributed plasma (top, $\kappa = 2$) and a thermal (Maxwellian) plasma (bottom). Equal electron and proton temperatures ($T_i = T_e$) and nonrelativistic thermal electron velocity $u_e = 10^{-3}$c are adopted.

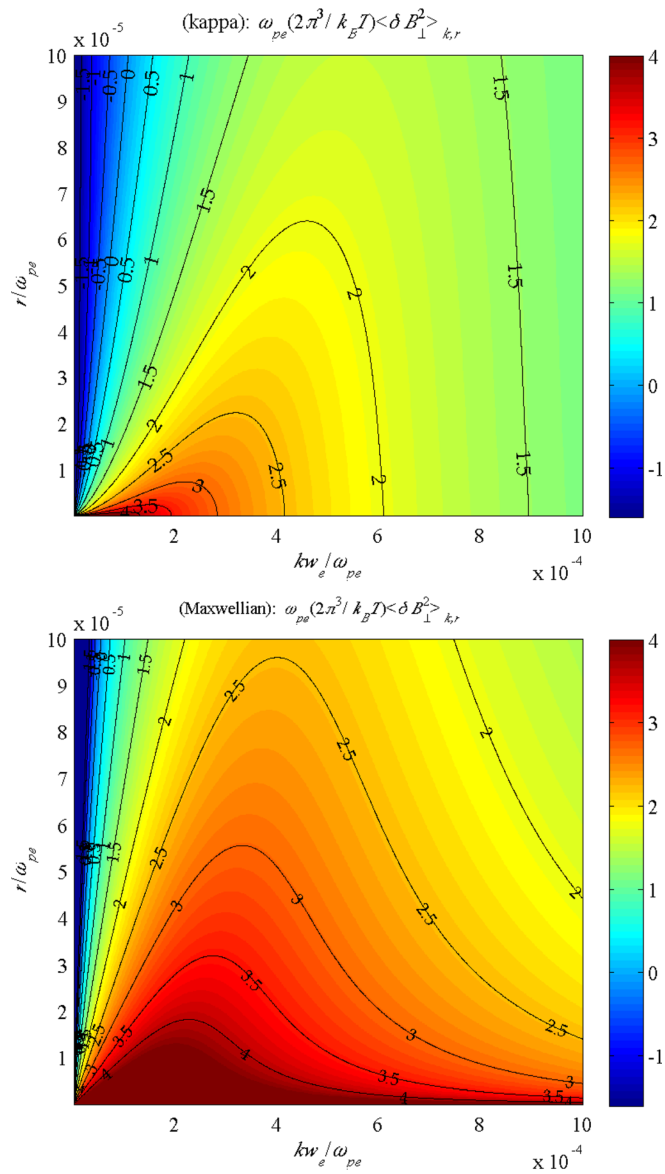

FIG. 5. Contour and color map plots of the spontaneously emitted weakly amplified magnetic field fluctuation in units of $k_BT_e/(2\pi^3)$ by contrast for a suprathermal Kappa-distributed plasma (top, $\kappa = 2$) and a thermal (Maxwellian) plasma (bottom). Equal electron and proton temperatures ($T_i = T_e$) and the nonrelativistic thermal electron velocity $u_e = 10^{-3}$c are adopted.

energy (thermal) electrons. This effect agrees with measurements of the electrostatic noise spectrum in a Kappa-distributed plasma.[16] The low-frequency fluctuations are not necessarily correlated with the collective ion-sound mode as this is heavily damped in isothermal ($T_e = T_i$) plasmas.[3]

The spontaneously emitted magnetic field fluctuations are displayed in Fig. 5 and show the maximum at zero frequency ($r = 0$), which is associated with the collective Weibel mode. In this case, the effect of suprathermal populations is opposite, lowering the level of magnetic field fluctuations (the same effect is observed for the collective fluctuations of the Weibel instability driven by a temperature anisotropy[11,23]).

III. CONCLUSIONS

In the first two papers of this series, the general expressions for the spontaneous fluctuations spectra (electric and magnetic field, charge and current densities) from uncorrelated plasma particles are derived and illustrated for a relativistic or nonrelativistic Maxwellian plasma close to thermal equilibrium. In this paper, the results are illustrated for the important case of a generalized model of plasma out of thermal equilibrium and described by the Kappa (power-law) particle distribution functions. In particular, the suprathermal fluctuations of weakly amplified modes and aperiodic modes are provided. The results obtained in the first paper for an equilibrium plasma are recovered only in the limit of a very large power index $\kappa \to \infty$.

Thus, it is shown for the first time the existing finite level of noncollective fluctuations, which are particularly important in the context of plasma fluctuations (collective or noncollective) as the most plausible mechanism of energy dissipation and transfer to suprathermal populations. Moreover, given the ubiquitous presence of these high-energy tail distributions in space plasmas, these results could significantly contribute to the new performances of plasma diagnosis based on highly sensitive measurements of quasithermal noise.

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APPENDIX A: DISPERSION FUNCTIONS FOR KAPPA DISTRIBUTED PLASMAS

In the nonrelativistic limit ($E \to 1$) and for an arbitrary distribution function, the dispersion functions (3) lead to the standard results

$$\Lambda_L = 1 + \frac{2\pi}{\omega}\sum_a \omega_{p,a}^2 \int_{-\infty}^{\infty} dv_\parallel v_\parallel \times \int_0^\infty dv_\perp \frac{v_\perp}{\omega - kv_\parallel}\frac{\partial F_a}{\partial v_\parallel} \tag{A1}$$

and

$$\Lambda_T = 1 - \frac{c^2k^2}{\omega^2} + \frac{\pi}{\omega^2}\sum_a \omega_{p,a}^2 \int_{-\infty}^{\infty} dv_\parallel \times \int_0^\infty dv_\perp v_\perp^2 \left[\frac{\partial F_a}{\partial v_\perp} + \frac{kv_\perp}{\omega - kv_\parallel}\frac{\partial F_a}{\partial v_\parallel}\right]. \tag{A2}$$

For an isotropic Kappa distribution function (5) we obtain for the longitudinal waves

$$\Lambda_L = 1 + \sum_a \frac{2\omega_{p,a}^2}{k^2 w_a^2}\left[\left(1 - \frac{1}{2\kappa}\right) + \frac{\omega}{kw_a} Z_\kappa\left(\frac{\omega}{kw_a}\right)\right] \tag{A3}$$

and for the transverse waves

$$\Lambda_T = 1 - \frac{c^2k^2}{\omega^2} + \sum_a \frac{\omega_{p,a}^2}{\omega k w_a} Z_\kappa^0\left(\frac{\omega}{kw_a}\right), \tag{A4}$$

in terms of the modified plasma dispersion functions, as it was derived for longitudinal modes[20,25] (also known as the "3D Kappa dispersion function")

$$Z_\kappa(f) = \frac{\pi^{-1/2}\Gamma(\kappa)}{\kappa^{1/2}\Gamma\left(\kappa - \frac{1}{2}\right)} \times \int_{-\infty}^{+\infty} \mathrm{d}t\, \frac{(1 + t^2/\kappa)^{-(\kappa-1)}}{t - f}, \quad \mathrm{Im}(f) > 0, \tag{A5}$$

and for the transverse waves[24,26] (also known as the "1D Kappa dispersion function")

$$Z_\kappa^0(f) = \frac{\pi^{-1/2}\Gamma(\kappa)}{\kappa^{1/2}\Gamma\left(\kappa - \frac{1}{2}\right)} \times \int_{-\infty}^{+\infty} \mathrm{d}t\, \frac{(1 + t^2/\kappa)^{-\kappa}}{t - f}, \quad \mathrm{Im}(f) > 0. \tag{A6}$$

Both of them approach the standard (Maxwellian) dispersion function (shown in the Appendix of Paper I) in the limit of a large $\kappa \to \infty$. We can use any form of the modified dispersion function in Eq. (A5) or Eq. (A6), as they are related by the transformation

$$Z_\kappa^0(f) = \frac{(\kappa - 1)^{3/2}}{(\kappa - 3/2)\kappa^{1/2}} Z_{\kappa-1}\left[\left(\frac{\kappa - 1}{\kappa}\right)^{1/2} f\right] = \left(1 + \frac{f^2}{\kappa}\right) Z_\kappa(f) + \frac{f}{\kappa}\left(1 - \frac{1}{2\kappa}\right), \tag{A7}$$

but the one providing the most simple expression of dispersion relation is of course recommended.

APPENDIX B: ASYMPTOTIC LIMITS OF THE MODIFIED DISPERSION FUNCTION

Because $\omega = r + \imath\gamma$ is a complex number we use the following simple notation for the argument of the dispersion function $f \equiv \omega/(kw) = x + \imath y$, where $x \equiv r/(kw)$ and

$y \equiv \gamma/(kw)$ are real numbers. The first modified plasma dispersion function for a representative value of $\kappa = 2$ has the following asymptotic expansions:[20]

$$\lim_{x\to 0} \Re Z_2(x+\imath y) \simeq \frac{-3(y^2+4\sqrt{2}y+10)x}{4(y+\sqrt{2})^4} + \frac{(3y^2+18\sqrt{2}y+70)x^3}{4(y+\sqrt{2})^6} + \cdots, \quad \text{(B1)}$$

$$\lim_{x\to 0} \Im Z_2(x+\imath y) \simeq \frac{3y^2+9\sqrt{2}y+16}{4(y+\sqrt{2})^3} - \frac{3(y^2+5\sqrt{2}y+16)x^2}{4(y+\sqrt{2})^5} + \cdots, \quad \text{(B2)}$$

$$\lim_{y\to 0} \Re Z_2(x+\imath y) \simeq -\frac{x(3x^4+20x^2+60)}{4(x^2+2)^3} + \frac{48\sqrt{2}xy}{(x^2+2)^4} + \cdots, \quad \text{(B3)}$$

$$\lim_{y\to 0} \Im Z_2(x+\imath y) \simeq \frac{8\sqrt{2}}{(x^2+2)^3} + \frac{3(x^6+10x^4+60x^2-40)y}{4(x^2+2)^4} + \cdots. \quad \text{(B4)}$$

Using these expansions we find

$$Z_2(x=0,y) \simeq \imath\, \frac{3y^2+9\sqrt{2}y+16}{4(y+\sqrt{2})^3}, \quad \text{(B5)}$$

$$Z_2(x,y=0) \simeq -\frac{x(3x^4+20x^2+60)}{4(x^2+2)^3} + \frac{\imath\, 8\sqrt{2}}{(x^2+2)^3}, \quad \text{(B6)}$$

and after the transformation (A7), we find for the second form of the modified dispersion function

$$Z_2^0(x=0,y) \simeq \imath\, \frac{y+2\sqrt{2}}{(y+\sqrt{2})^2}, \quad \text{(B7)}$$

$$Z_2^0(x,y=0) \simeq -\frac{x(x^2+6)}{(x^2+2)^2} + \imath\, \frac{4\sqrt{2}}{(x^2+2)^2}. \quad \text{(B8)}$$

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